Connecting Singular, GAP, Polymake, and Gfan

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Integrating new features into Singular

Investigate mathematical questions combining

- Algebraic Geometry
- Group Theory
- Number Theory
- Convex Geometry
- Tropical Geometry

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For system developers:

- Integrating custom C++ code into SINGULAR.

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Application:

- Computing the GIT-fan.

Using Polymake in Singular

Example

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> LIB "polymake.so";
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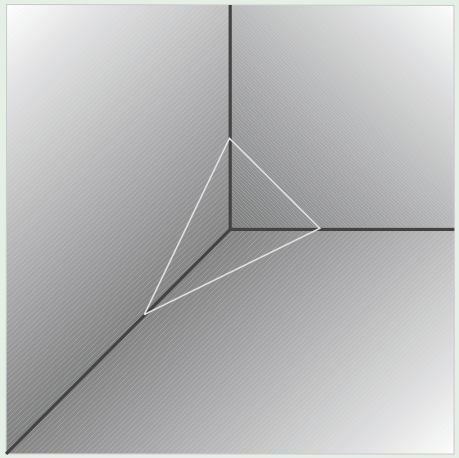
> ring r = 0,(x,y,z),dp; poly g = x3+y3+1;
> polytope p = newtonPolytope(g);
> fan F = normalFan(p); F;
   RAYS:
   -1 -1 0 #0
   0 1 0   #1
   1 0 0   #2
   MAXIMAL_CONES:
   {0 1} #Dimension 3
   {0 2}
   {1 2}
```



Integrating custom C++ code

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// To make the function plus available in the interpreter add the following to
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// To make the function plus available in the interpreter add the following to
// extra.cc or to your own shared library:
BOOLEAN plus(leftv res, leftv args)
{
        int n = (int)(long)args->Data();
        int m = (int)(long)args->next->Data();
        res->rtyp = INT_CMD;
        res->data = (void*) n+m;
        return TRUE;
}
```

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        return TRUE;
}
void add_setup()
{
        iiAddCproc("","add",FALSE,add);
}
```

In this way GFAN and POLYMAKE types and functions were integrated into SINGULAR.

User defined data types in Singular

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      QQ t;
      t.num = n; t.den = d;
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> proc addQQ(QQ r, QQ s){
      return(frac(r.num*s.den + s.num*r.den, r.den*s.den));}
```

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      return(frac(r.num*s.den + s.num*r.den, r.den*s.den));}
> system("install","QQ","+",addQQ,2);
> frac(2,3)+frac(3,5);
  19/15
```

Application: Computing the GIT-fan

$J \subset K[x_1, ..., x_n]$ homogeneous w.r.t. $Q = (q_{ij}) \in \mathbb{Q}^{r \times n}$ and $X = V(J)$.

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GIT-fan describes the variation of good quotients $U /\!\!/ \mathbb{T}^r$ with $U \subseteq X$.

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Algorithm [Keicher, 2012]:

Face $\sigma \subseteq \gamma := \mathbb{R}^n_{\geq 0}$ is **J-face** if the $\mathbb{T}^n$-orbit of σ intersects X.

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Algorithm [Keicher, 2012]:

Face $\sigma \subseteq \gamma := \mathbb{R}^n_{\geq 0}$ is **J-face** if the $\mathbb{T}^n$-orbit of σ intersects X.

$$\Omega_J := \{Q(\sigma) \mid \sigma \text{ is } J\text{-face}\}$$

GIT-fan $\Lambda(J, Q)$ consists of all **GIT-chambers**

$$\lambda(w) = \bigcap_{\substack{\vartheta \in \Omega_J \\ w \in \vartheta}} \vartheta \qquad \text{for } w \in Q(\gamma).$$

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We compute the GIT-fan of $\mathbb{G}(2,4)$:

Example

```
> LIB "gitfan.lib";
> ring R = 0,x(1..6),dp;
> ideal I = x(1)*x(6) - x(2)*x(5) + x(3)*x(4);
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                   0,1,0,1,0,1,
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                   0,1,0,1,0,1,
                   0,0,1,0,1,1,
> fan F = gitFan(I, Q);
> rays(F);
   0 0 1
   0 1 0
   0 1 1
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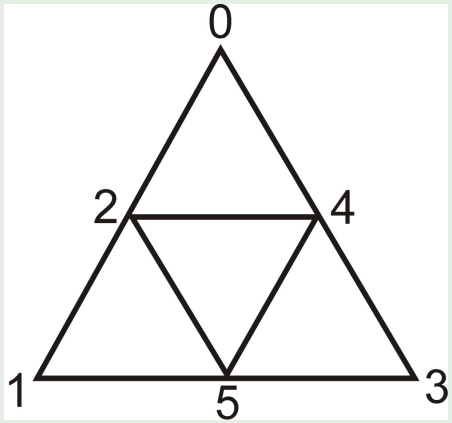

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> LIB("parallel.lib","random.lib");
> ring R = 0,x(1..4),dp;
> ideal I = randomid(maxideal(3),3,100);
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> LIB("parallel.lib","random.lib");
> ring R = 0,x(1..4),dp;
> ideal I = randomid(maxideal(3),3,100);
> proc sizeStd(ideal I, string monord){
      def R = basering; list RL = ringlist(R);
      RL[3][1][1] = monord; def S = ring(RL); setring(S);
      return(size(std(imap(R,I))));}
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> list commands = "sizeStd","sizeStd";
> list args = list(I,"lp"),list(I,"dp");
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> list commands = "sizeStd","sizeStd";
> list args = list(I,"lp"),list(I,"dp");
> parallelWaitFirst(commands, args);
   [1] empty list
   [2] 11
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> parallelWaitAll(commands, args);
   [1] 55
   [2] 11
```